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Three Degree of Freedom Control of Inverted Pendulum by using Repulsive Magnetic Levitation

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Abstract: An inverted pendulum is a pendulum which has its mass above its pivot point. Inverted Pendulum is a classical problem in dynamics and control theory and is widely used as a benchmark for testing control algorithms (PID controllers, neural networks, fuzzy control, genetic algorithms etc.). Variations on this problem include multiple links allowing the motion of the cart to be commanded while maintaining the pendulum and balancing the cart- pendulum system on see- saw. The inverted pendulum is related to rocket or missile guidance, where the centre of gravity is located behind the canter of drag causing aerodynamic instability. The understanding of similar problem can be shown by balancing an upturned broomstick on the end of one's finger is a simple demonstration and the problem is solved in the technology of the seaway Position, self balancing transportation device. So, here the entire above problem can be solved by using levitation system. In this levitation system three magnets are used in equilateral triangle for giving the force to inverted pendulum in 3-dimension for balancing the pendulum.

***Index Terms*- Inverted pendulum, Repulsive Magnetic Levitation, centre of gravity**

I. Introduction

Inverted pendulum is often implemented with the pivot point mounted on a cart that can move horizontally [4]. Most application limits the pendulum to two degree of freedom by affixing the pole to an axis of rotation [7]. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright. This can be done either by applying torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to and thereby generating a net torque on the pendulum, or by oscillating the pivot point vertically[5]. If the driving point moves in simple harmonic motion, the pendulum's motion is described by the Mathieu equation [10]. Our target is to balance the pendulum in 3-dimensional space like a conical plane [9]. Here we place three magnets in three corners of a equilateral triangle. The inverted pendulum is placed at the orthocentre of equilateral triangle. By the help of electromagnetic force produced by three magnets, the inverted pendulum can be made stable in upright position by levitation system.

II. Mathematical Calculations

From the Figure 1, xyz is the equilateral triangle in which on x point of equilateral triangle one magnet M1 is placed and on y point, magnet M2 is placed and on z point,magnet M3 is placed. All the three magnets are placed by placing pole on the cone of the equilateral triangle and the magnets have hemispherical faces pointed towards the pendulum which is in orthocentre of equilateral triangle.

Here the three axes X,Y and Z are present in which for every axes driver circuit,controller and current driver are present.Here driver circuit is used for giving the balance force to the inverted pendulum,controller is assigned for controlling the balance force given by the three magnets so that pendulum can be controlled in satble position and current driver is assigned for giving the current im1,im2 and im3 which is the component of force delivered by three magnets. Here position of the pendulum is sence by useing [1] P1,P2,P3 position sensor.

Using the fundamental principle of dynamics, the behaviour of the ferromagnetic pendulum is given by the following electromechanical equation [2].

$$m\frac{d^2y}{dx^2} = mg + f(x,i) \tag{1}$$

Where $f(x, i)$ is the magnetic control force given by

$$f(x,i) = -k\frac{i^2}{x^2} \tag{2}$$

In equation (1) m (mass) of the levitated pendulum, g (acceleration due to gravity),x is the distance of the centre of mass of the pendulum from the electromagnet, and in equation (2) i(current through the coil). In addition to that, k is a constant related to the mutual inductance of the

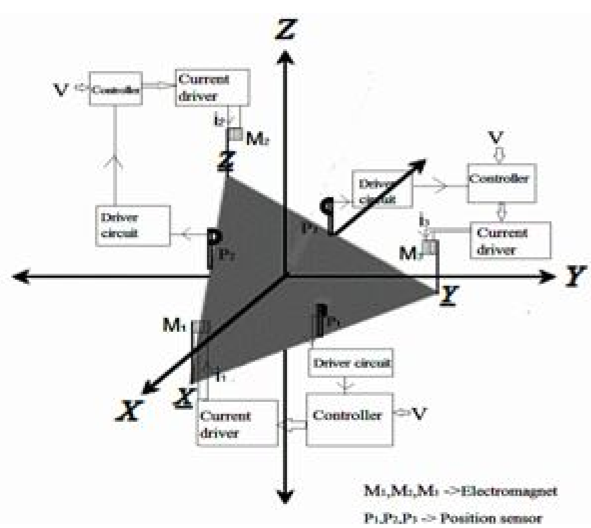

Fig. 1. Model of lavitation system

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pendulum and coupling coefficients [2]. So f1(x1, im1), f2(x2, im2) and f3(x3, im3) will be the three forces which will generate by three magnetic fields M1, M2 & M3.

II. Magnetic Pole Placment

In the Figure 2, magnets M1, M2 and M3 are the position, where pole will be placed, which is on the cone of euilateral triangle which have hemispherical faces pointing towards the inverted pendulum and all the three magnets and position of centre of mass of the pendulum all lie on same level. Here Z axis is showing the direction of inverted pendulum which is placed in orthocentre of equilateral triangle. S is the axis of direction for showing the direction of inverted pendulum in inclined position. The forces [8] applied by the three magnets on the inverted pendulum is balanced so that inverted pendulum can be stable in any position and according to the position of inverted pendulum the forces of all the three magnets are adjusted [9]. And always follow equation (3).

$$f1(x1, im1) + f2(x2, im2) + f3(x3, im3) = 0 \quad (3)$$

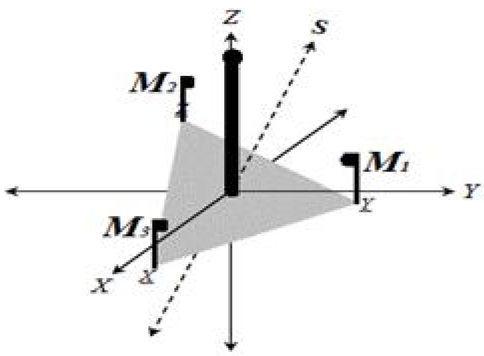

Fig. 2. Position of magnet(M1, M2, M3)

VI. Different Situation Of Pendulum

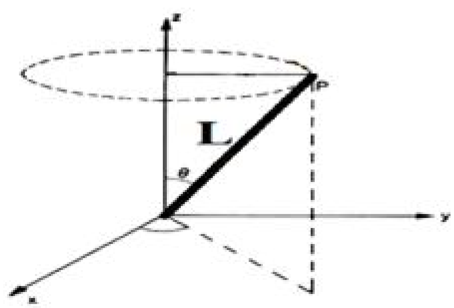

Fig. 3. Different position of pandulum

Here in the Fig.3, x, y and z are the axes of 3 dimension[11].L is the length of the inverted pendulum and P is the top point of inverted pendulum.The circle around the Z axis is showing the rotation of inverted pendulum and the angle between the Z axis and the pendulum. Here the stability of pendulum is cosidered on the basis of centre of gravity. So if the pendulum is completely in vertical position then centre of gravity is located at the orthocentre of equilateral triangle otherwise the centre of gravity of the pendulum is displaced from the orthocentre of the triangle .

A. Condition Of Verical Position

Figure.4 shows the top view of the inverted pendulum in which the inverted pendulum lie on the orthocentre of the equilateral triangle.Here, inverted pendulum is in vertical position,so the centre of gravity of the pendulum lies on the orthocentre of equilateral triangle which can be seen in figure.4 by considering the top view [11].where Fm1 is the force which is applied by the magnet M1, Fm2 is the force which is applied by the magnet M2 and Fm3 is the force which is applied by the magnet M3 in which all the three forces coincide in the orthocentre of the equilateral triangle For balancing the forces Fm1,Fm2 and Fm3, all the three forces can be aligned in same axis so that we can get the condition for stable and balanced position of the pendulum[6].So

Fm3 = Fm1 cos60 + Fm2 cos 60

and we know that cos60 =1/2

So, Fm3 = ½(Fm1+Fm2)

So from above equations we can say that the condition for stable condition is- Fm3 = Fm2 = Fm1

So from eq. (2), $f1(x1, im1) = f2(x2, im2) = f3(x3, im3)$

Thus, im1 = im2 = im3

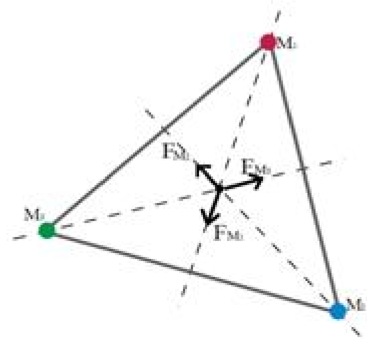

Fig. 4. Top view of the system in equbliriam condition

B. Condition Of Inclined Position

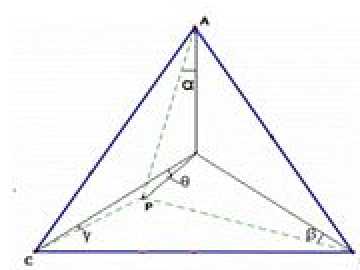

Fig. 5. Top view of the system when pendulum is inclinned

In the above figure,there is also top view of the inverted pendulum but here the inverted pendulum is in inclined position so that the centre of gravity of inverted pendulum is displaced from the orthocentre of the equilateral triangle which can be seen from the figure 5. The forces Fm1 is displaced from orthocentre by an angle "α", force Fm2 is displaced from orthocentre by an angle of "β" and Fm3 is displaced from orthocentre by an angle of "γ".All the three forces meet at a point "P".Let the point "P" is displaced from orthocentre by an angle of "θ" with respect to the line joining the pont "C" and orthocentre[9]. So, for getting the balance condition of all the three forces at point "P" we have to allign all three

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forces in one axiz.So, after doing that we get

Fm3 = Fm1cos(60+β-γ)+ Fm2cos(60-α-β) (3)

and 2nd condition for equilibrium is ......

Fm1 sin(60+β-γ) = Fm2 sin(60-α-β) (4)

ƒ3(x3, im3) = ƒ1(x1, im1) cos(60+β-γ) + ƒ2(x2, im2)cos(60-α-β) (5)

From eq. 3 &4

ƒ3(x3, im3)=ƒ1(x1, im1) [cos(60+β-γ)+tan(60-α-β) * sin(60+β-γ)] (6)

So im3= im1[cos(60+β-γ)+tan(60-α-β) * sin(60+β-γ)]

im2= im1[sin(60+β-γ) /sin(60-α-β)] (7)

So by adjusting the current according to eq.7, can get equilibrium position on that particular position or on that particular angle.

But for reach the vertical position, this is possible only by adjusting the force so after some time all force are equal.

ƒ1(x1, im1) = ƒ2(x2, im2) =ƒ3(x3, im3)

And α = β = γ

By doing this we can bring the point "P" to the othocentre so that equilibrium position can be reached so inverted pendulum can become stable in any position of inclination.

V. Operation Of Magnetic Levitation

In magnetic Levitation Magnetic pressure is used to counteract the effects of the gravitational and any other acceleration [3].

Here are three systems on a single platform and each system consists of two controllers: a lead compensation controller with the possibility to change its characteristics (i.e., bandwidth, gain) and a controller driven by software[13].

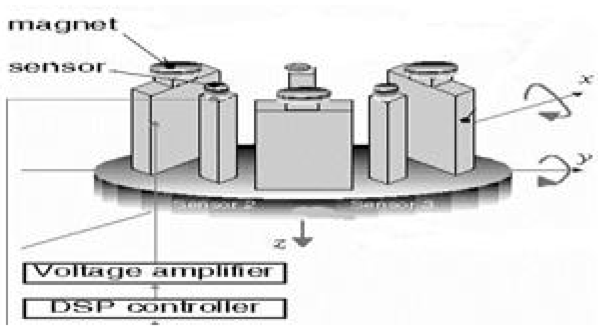

Fig. 6. Practical model of this magnetic sus

The photo-sensor (infrared-based) measures the position of centre of mass of pendulum. It provides measurement of the distance of the centre of mass of pendulum from the electromagnet by providing a voltage *Vsensor* such that:

Vsensor = -γ(*X* - *Xo*),γ > 0

Where *X*o is the nominal operating point.

Therefore, since *Vsensor* = 0 for *X* = *X*o,

We get ΔV*sensor* = -γ Δ*X*

The current *I* through the electromagnet is controlled by an inner-loop and is related to the voltage controller output *U* by the expression I = 0.15*U* + *I*o

Where *I*o is the nominal current corresponding to the nominal operating position *X*o

Therefore, the relation between current variations and control variations is: Δ*I* = 0.15Δ*U*

The equations for electromagnetic force produce inside the coil system are:- $B = \mu * N * a^2 * I/2(a^2 + z^2)^{3/2}$

Where μ=permeability of vacuum, N = turns,

I =currents (amperes), a = radius (metres)

z = axial distance from coil (metres)

Let the current Im1, Im2 and Im3 are the currents which are the component of the forces generated by the three magnets M1, M2 and M3 respectively so that force Fm1, Fm2 and Fm3 are generated by these magnets. These forces are as follows:

$Fm1 = \mu * Im1^2 * x/2 * \pi * d$

$Fm2 = \mu * Im2^2 * x/2 * \pi * d$

$Fm3 = \mu * Im3^2 * x/ 2 * \pi * d$

Where, Fm1, Fm2 & Fm3 are forces applied by magnet M1,M2 & M3.and Im1,Im2 and Im3 are the currents of three magnets[13][3], μ = Permeability, x = distance between magnet and Pendulum (centre of mass), d = radius of magnets

Here voltage output from the photo-sensor (infrared-based) with the position of the pendulum while the pendulum is in the effective region of the position sensor system. This position signal is acquired by the controller. The control instruction will be sent to the current driver by the controller after control algorithm. The electromagnetic force to the pendulum will follow with the change of the current of the electromagnet coil which is supplied by the current driver.

A. Stability Of Magnetic Levitation

It can be said that when forces are in equilibrium or balanced with each other then inverted pendulum can remain in stable position.Stability can be defined in two types one is static and other is dynamic stability.Static Stability means that any small displacement away from a stable equilibrium causes a net force to push it back to the equilibrium point. Earns haw's theorem proved conclusively that it is not possible to levitate stably using only static, macroscopic, paramagnetic fields. However, several possibilities exist to make levitation viable, for example, the use of electronic stabilization or diamagnetic materials.

Dynamic Stability occurs when the levitation system is able to damp out any vibration-like motion that may occur. Magnetic fields are conservative forces and therefore in e have no built-in damping, and in practice many of the levitation schemes are under-damped and in some cases negatively damped. This can permit vibration modes to exist that can cause the item to leave the stable region [12].

B. Phase – Lead Compensated Controller For Magnetically Levitated System

The simplest way to stabilize the system is to use the phase-lead compensated controller to cancel the unstable pole. The necessary pole required for the phase-lead compensated controller is placed deeper into the left hand plane. This will minimize the impact of the pole of the compensated controller on the root-locus [4].

Consider the compensator placed in the feed forward path is called a cascaded or Phase-Lead Compensator. Shown in fig.7. Where G1(s) = the open-loop magnetic levitation

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system transfer function.

Gc(s) = the open-loop compensator transfer function is used to adjust the system dynamics favourable without affecting the steady-state error, H(s) = 1,feedback path gain. Transfer function of the system:

$$G(s) = \frac{-2CIo\alpha/mL1Yo^2}{\left(s+\frac{R}{L1}\right)(s^2-2CIo^2/mYo^3}$$

Shown in fig.8.

Where Yo= Equilibrium distance, Io=Equilibrium Current, m= projected mass, C= force constant, R= coil resistance, L1= coil inductance, α = sensor gain

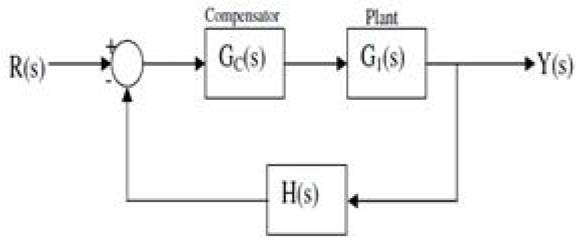

Fig. 7. Magnetic Levitation System with Phase-Lead Compensator

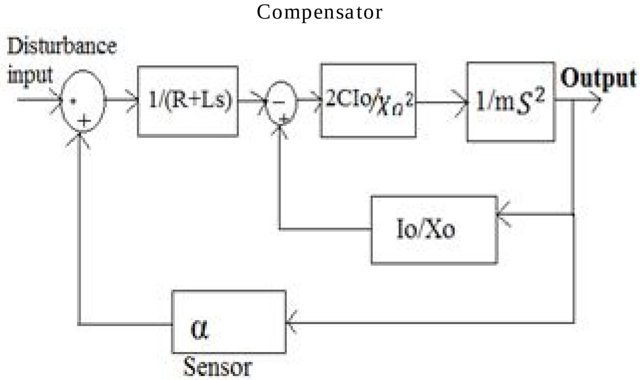

Fig. 8. Transfer function Block Diagram

Conclusion

The overall conclusion of our topic is that the inverted pendulum can be stable by the levitation system which is more flexible. Here the three magnets are used in three corner of equilateral triangle which produces balanced force towards the inverted pendulum to make pendulum stable. So finally we can say that when the forces generated by the three magnets are balanced or in equilibrium considering the position of centre of gravity then the inverted pendulum will remain in stable at any position.

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